Assessing the Mean Neuronal Firing Rate Information Hypothesis via Mutual Information

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J.N. Andrews Honors Program Honors Thesis Final Oral Presentation

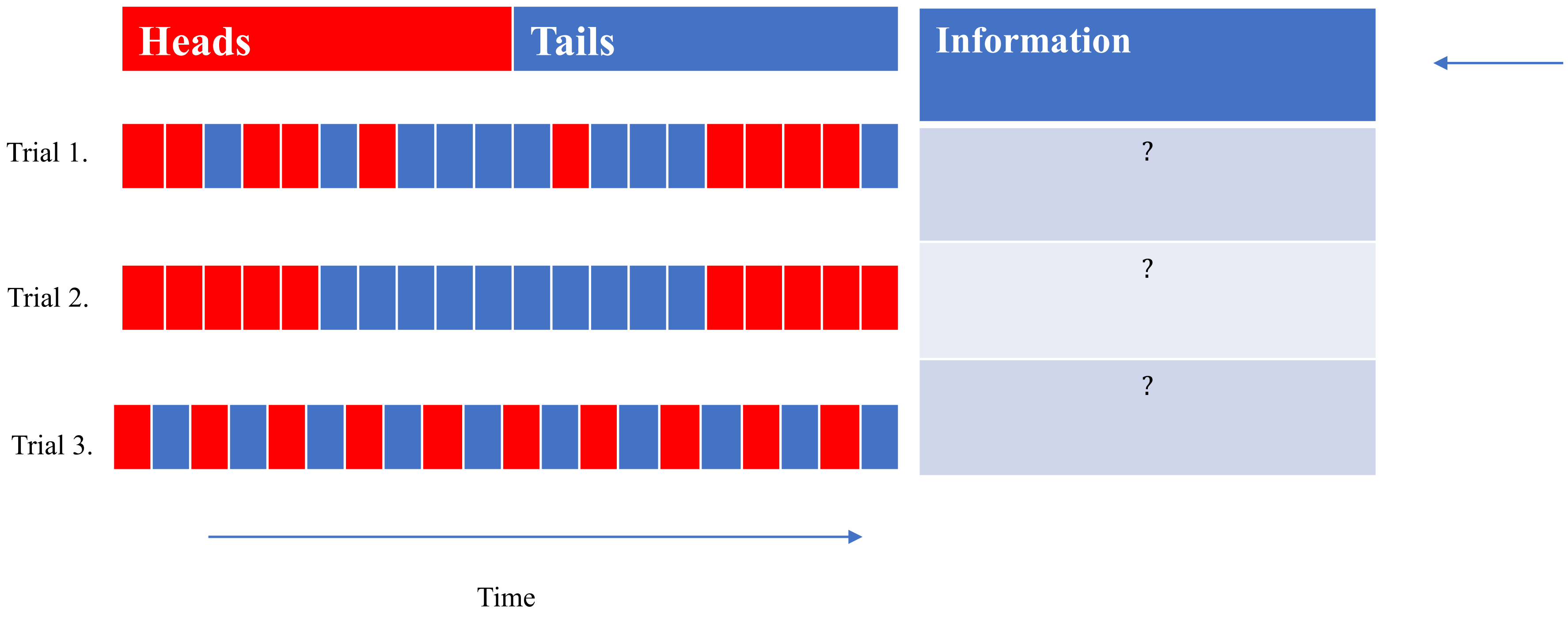
Thursday, November 15th, 2018 6:00 PM

Andrews University

J.N. Andrews Honors Program

Introduction: An Illustration

- Flipping a coin experiment
  - Same number of heads and tails (identical distributions)
  - 10 tails, 10 heads in each trial



Which trial contains the most information, pattern, or structure?

Introduction: An Illustration

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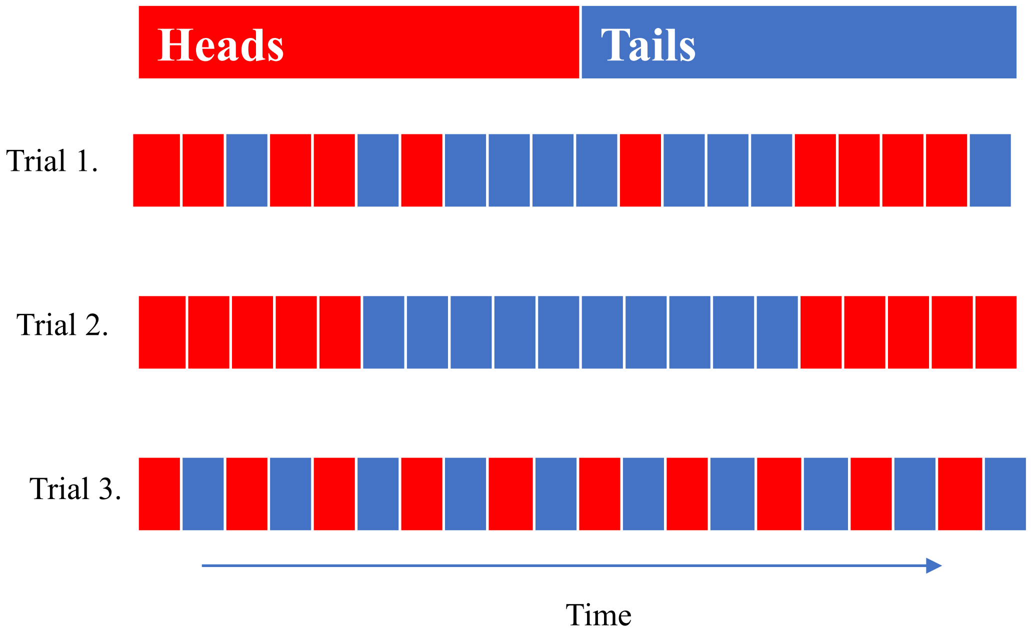
  - 10 tails, 10 heads in each trial



Information
0.008
0.81
1

Relative structure or information

Project Context and Goal:

- Context in Neuroscience
    - The goal: understanding how neurons encode information
    - How to get to this goal? First quantify neuron signal information content
    - Well—accepted parameter encoding information for sensory and motor neurons is the mean neuronal firing rate[1] (MNFR), described by[2]:

$$MNFR = \frac{\sum spikes}{time\ interval}$$

    - Stein and colleagues have suggested MNFR does not account for all neuron signal information content[3]
    - null hypothesis: that the MNFR encodes all neuron signal information content

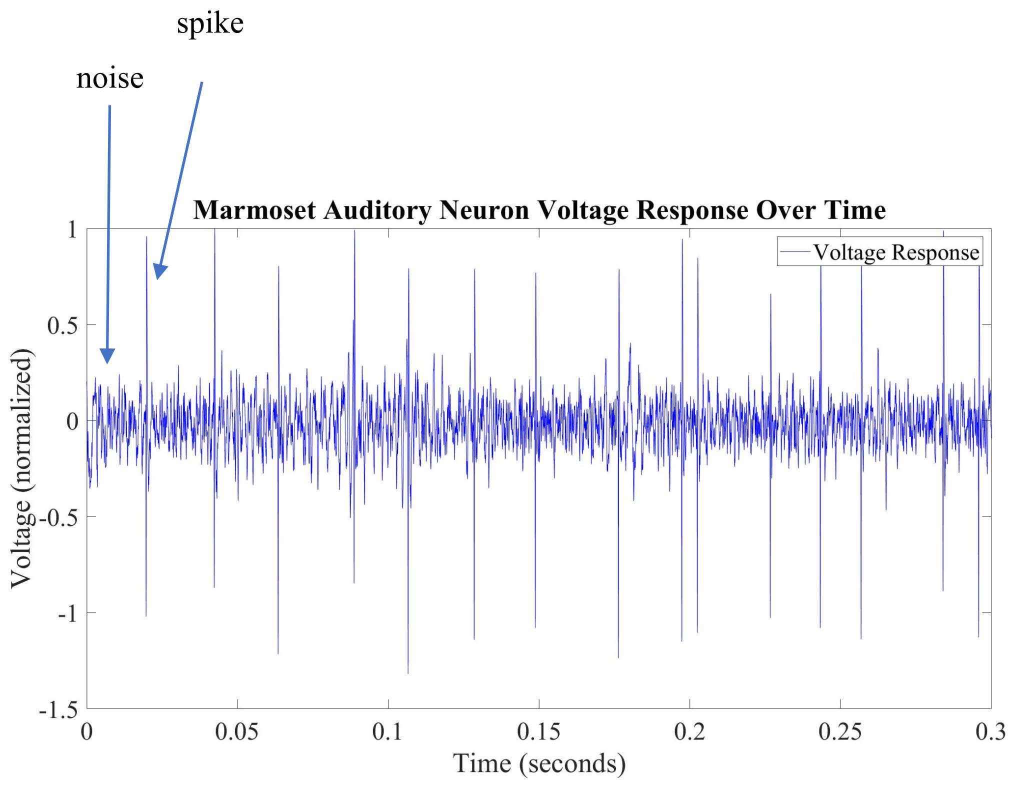


Figure 1. Spike train from single Marmoset monkey auditory neuron with a MNFR of 0.53 spikes/second

- Goal
    - Test null hypothesis via comparing actual signal data information content versus surrogate or simulated signal data information content, using mutual information as information metric

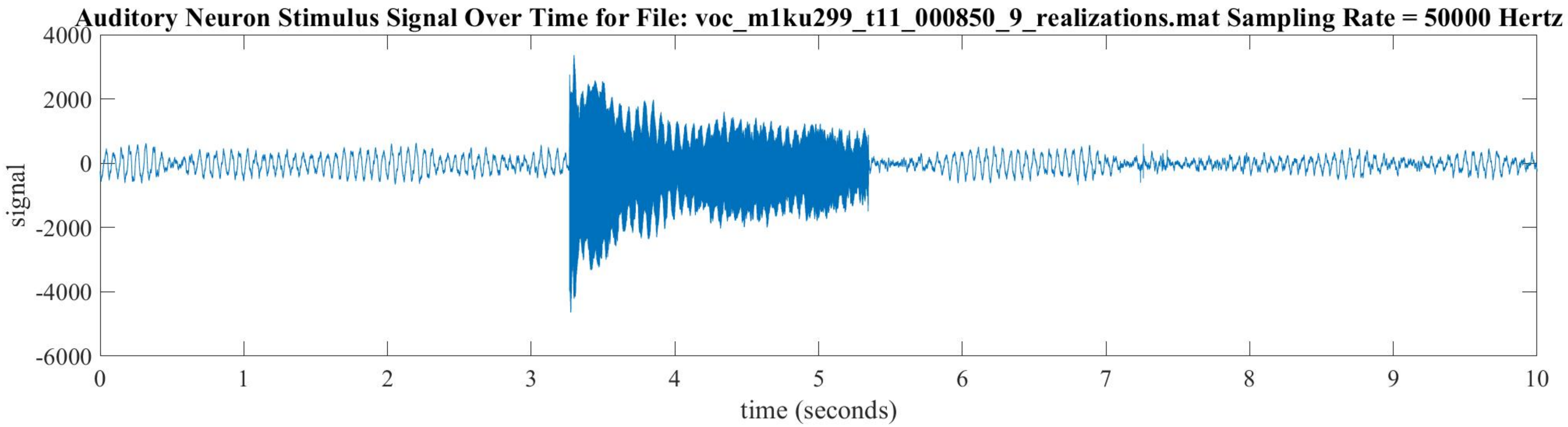
1. Rieke, F. Warland, D. Steeninck, R. Bialek, W. *Spikes Exploring the Neural Code*. MIT Press 1999.
2. Borst, A. Juergen, H. “Effects of Mean Firing on Neural Information Rate.” *Journal of Computational Neuroscience*. 10, 213-221, 2001
3. Stein, R. Gossen, R. Jones, K. “Neuronal Variability Noise or Part of the Signal?” *Nature Reviews: Neuroscience*. Vol 6. May 2005. P. 390-7.

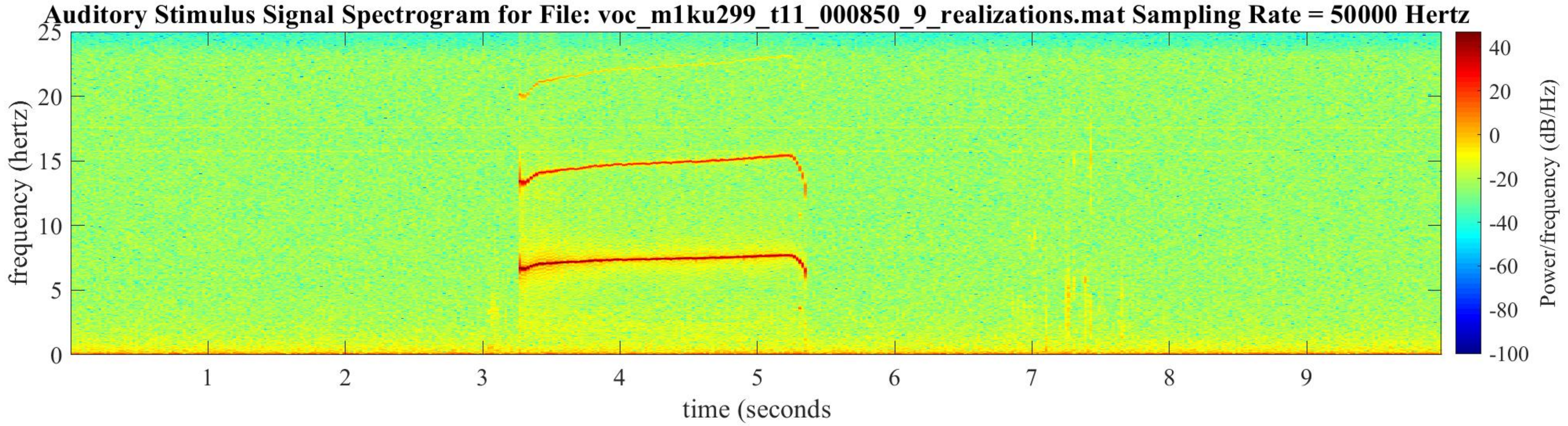
Data Background and Computation Tools

- Data recorded at the John Hopkins Laboratory of Auditory Neurophysiology with National Science Foundation (NSF) funding, ensuring ethical recording procedures
- Analysis performed in MATLAB, a numerical—based computing program
- Data
  - Data comes in pairs: stimulus and response files
  - 35 pairs => 35 data files to analyze
  - 10 seconds data / file

Auditory Stimulus

Phee vocalization example:

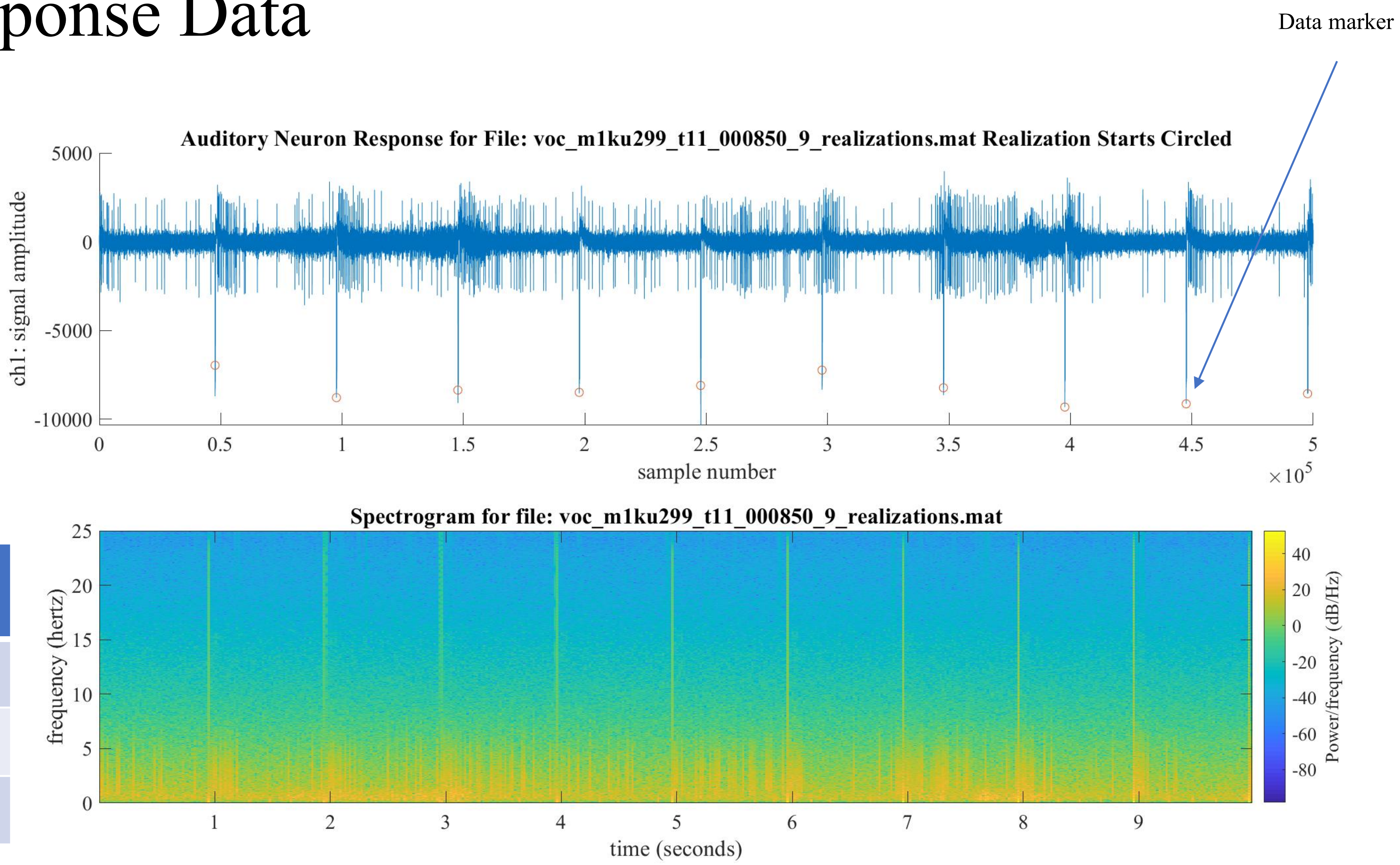




Neuronal Response Data

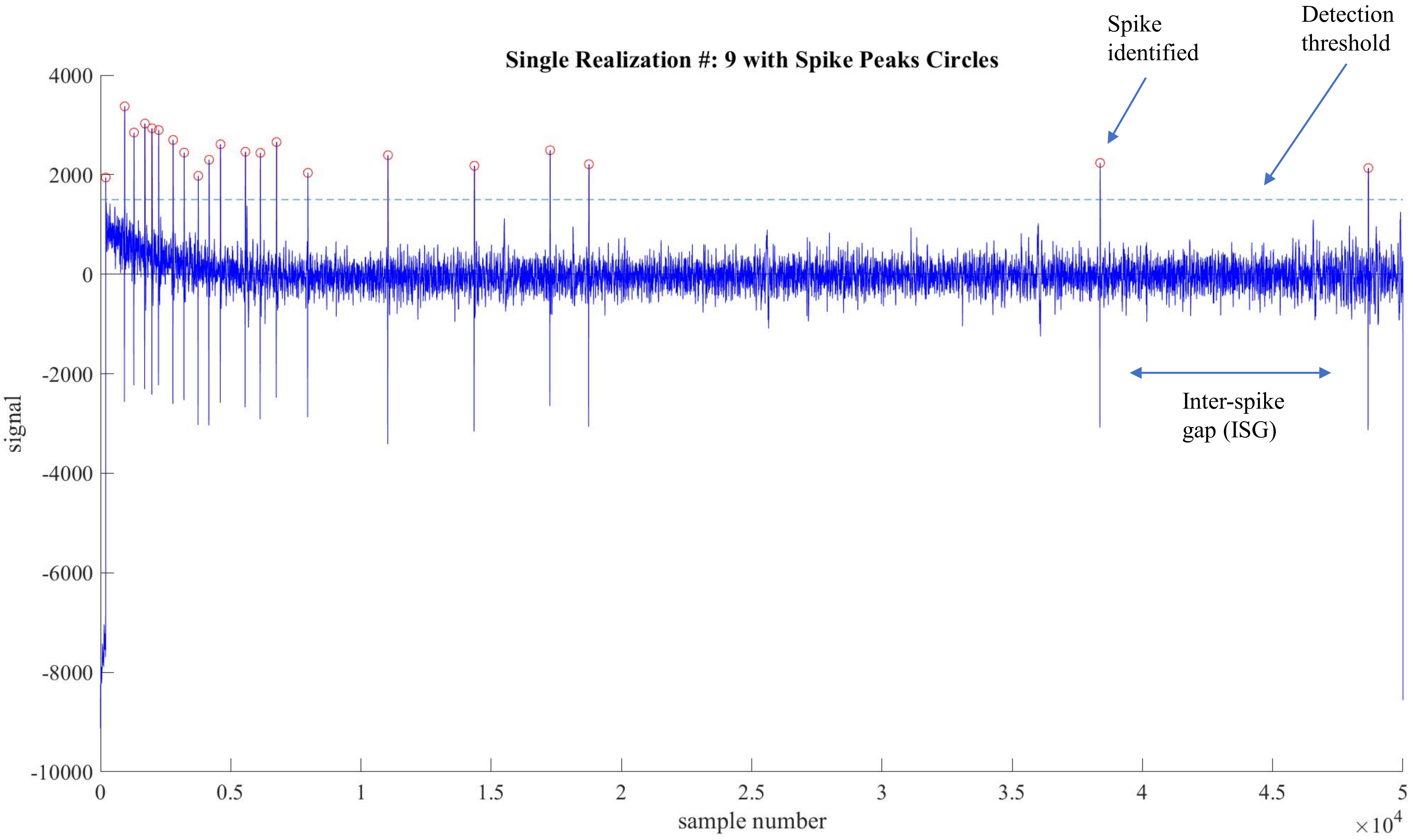
- Circled data markers between (-6000 and -8000) denote new instance of stimulus application, defined as a new ***realization***
- Sampling frequency = 50K Hz

Number of files	Number of realizations
23	> 3
10	3
2	9

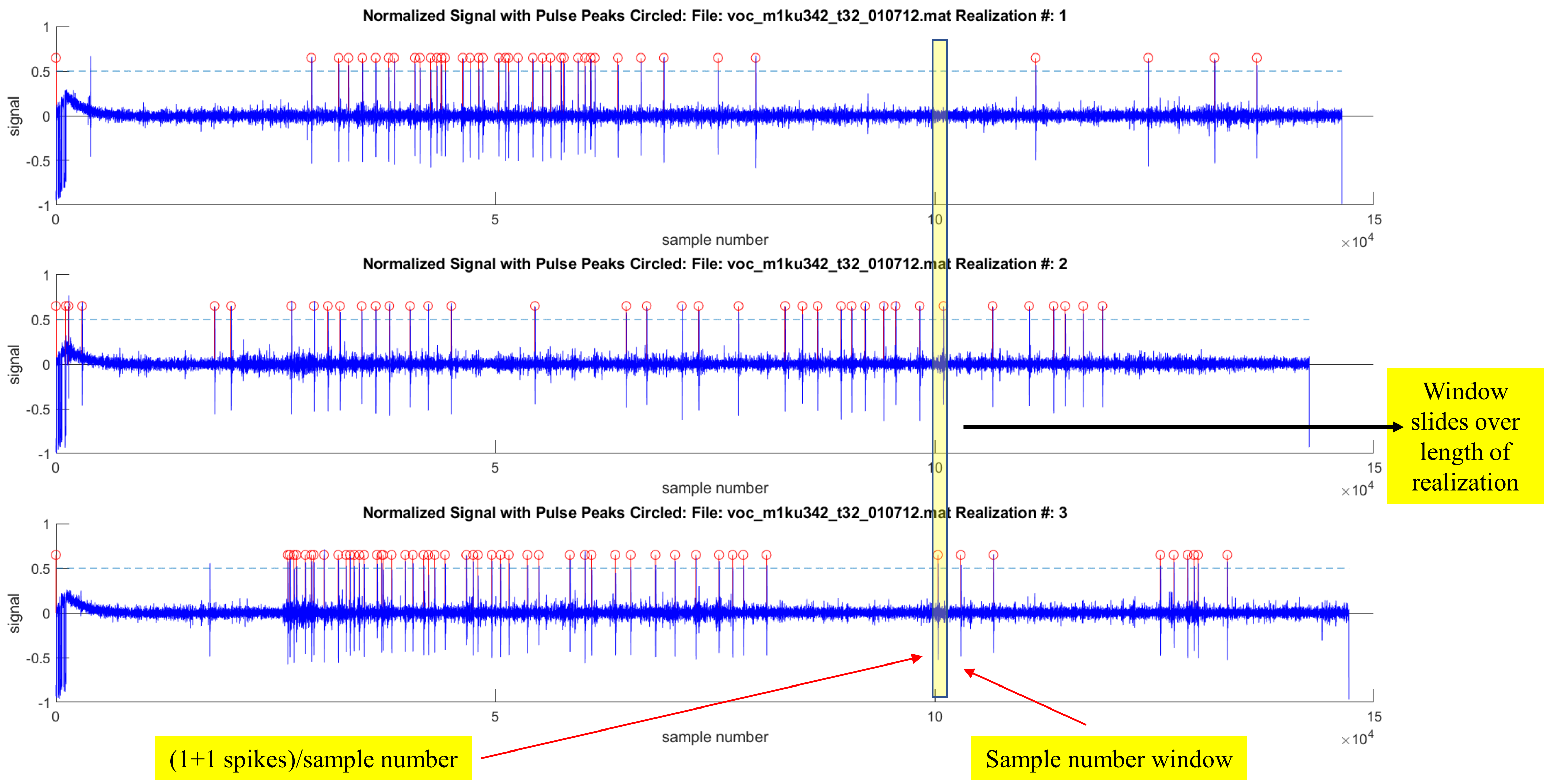


Spike Detection and Inter-Spike Gap (ISG) Time Series

- Detection parameters
  - Minimum spike separation
  - Minimum spike height
- High signal to noise (SNR) data allowed for constant threshold detection
- Distance between adjoining spikes calculated as the ISG, resulting in a time series of events

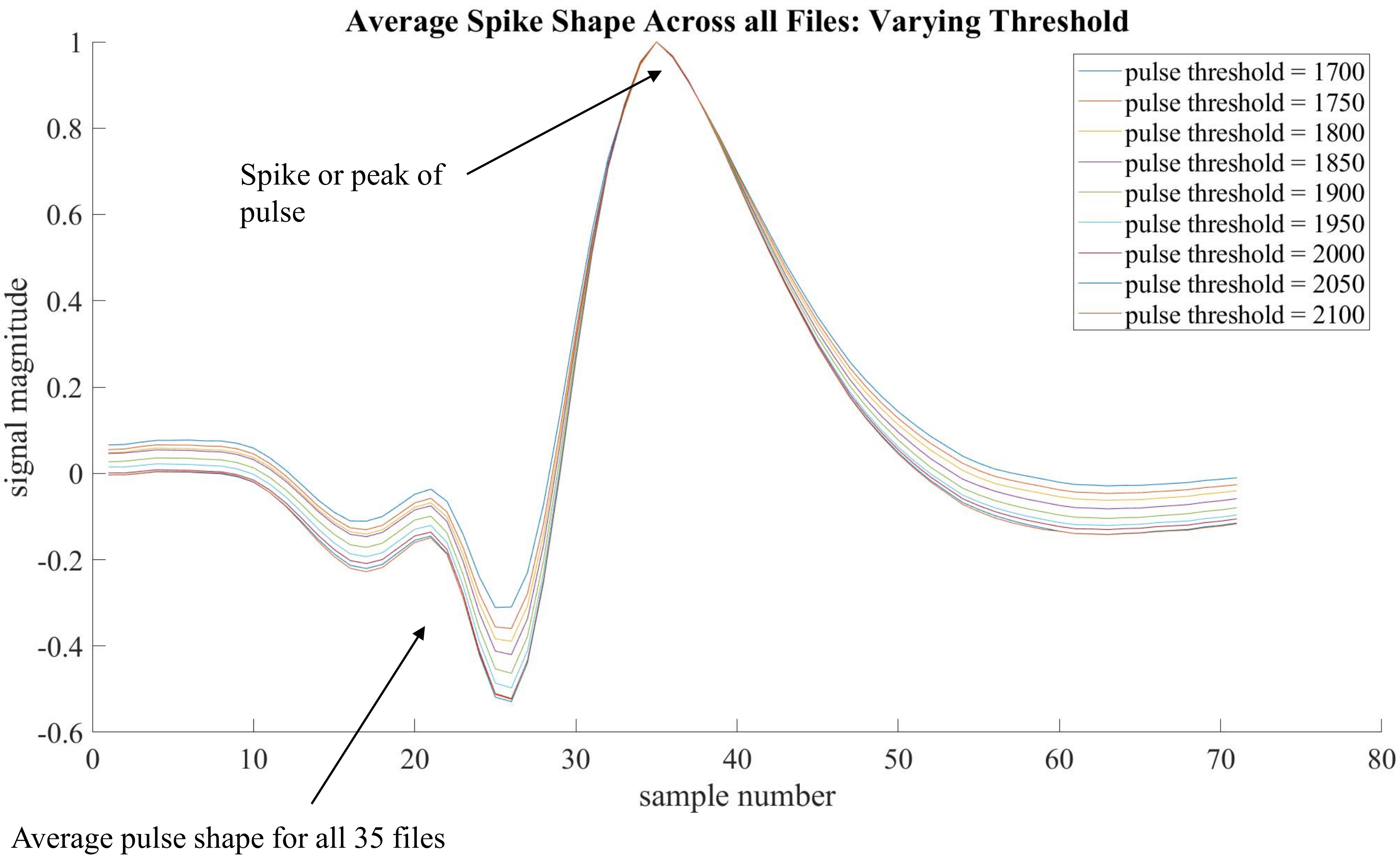


Summing Spikes Per Sample Number Over All Realizations



Pulse Shape: Binary Encoding of ISG Time Series

- Consistent pulse shape allowed for binary representation of presence of spikes
    - 0 = no spike
    - 1 = spike
- Benefits of binary form
    - Signal magnitudes are normalized
    - Computationally simpler



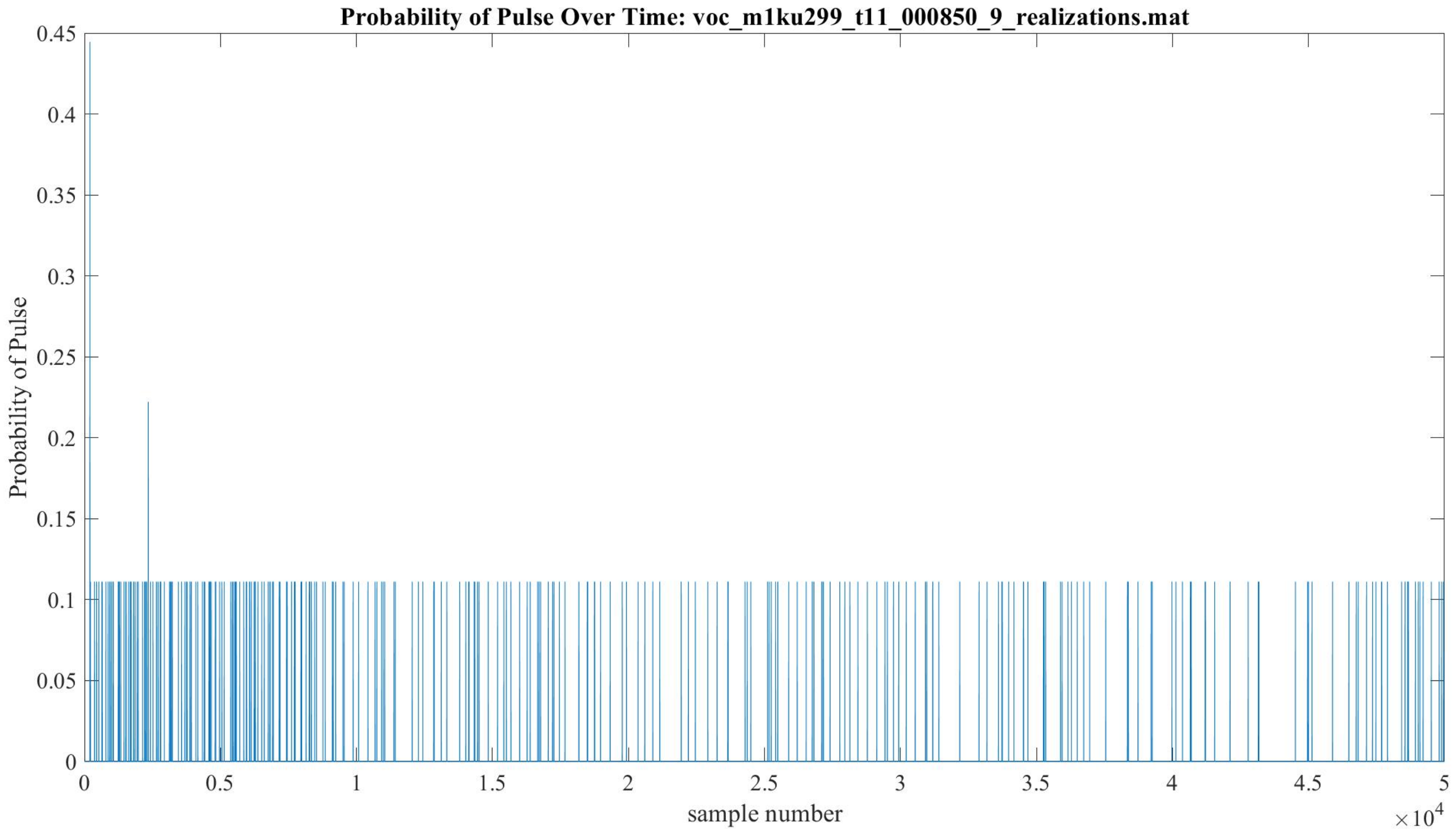
Realization Data Limitation

- Resolution of probability of a pulse (RPP) at a given instance in time (sample number) is given by:

$$RPP = \frac{1}{(Number\ Realizations\ /\ file)}$$

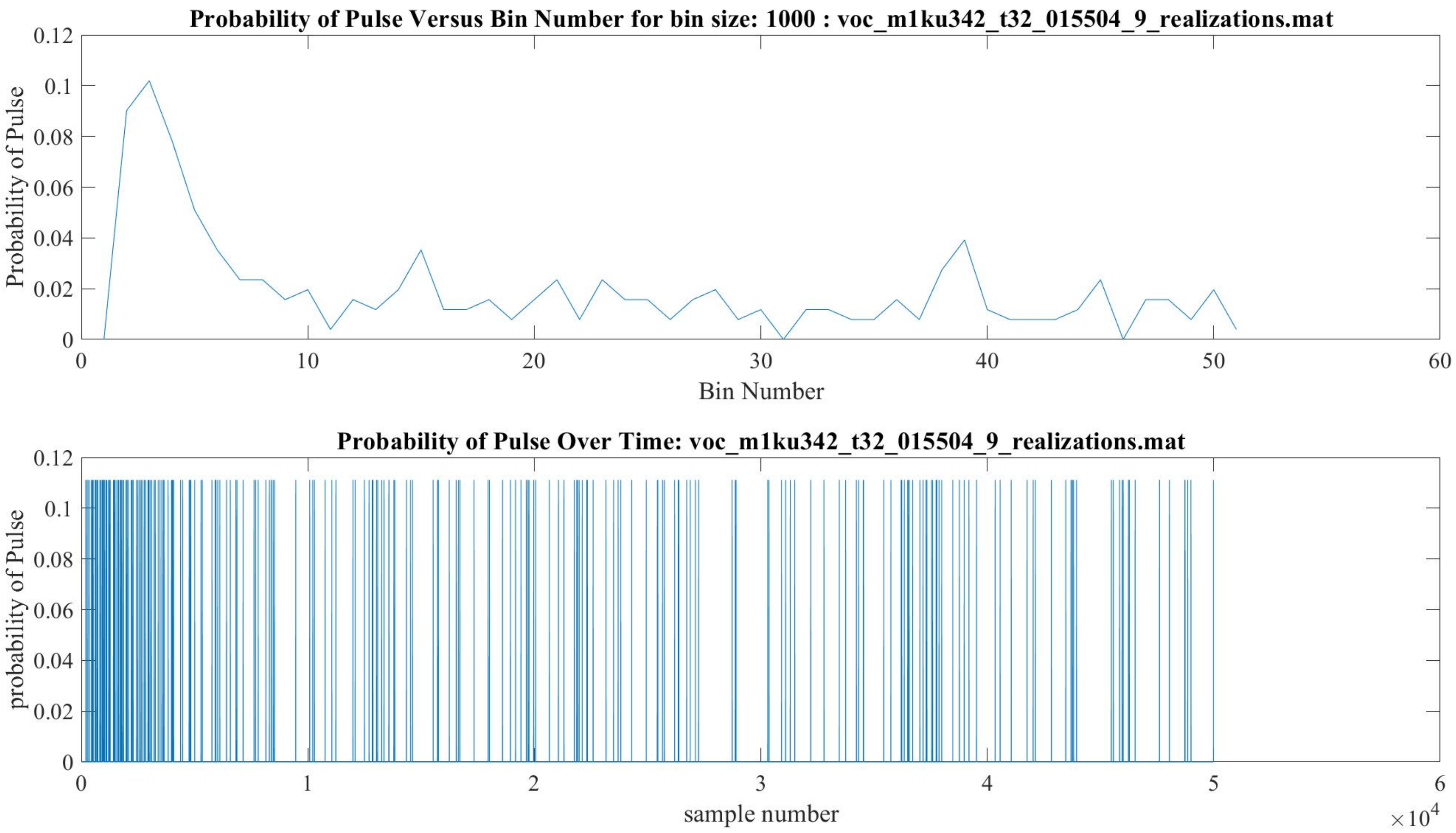
- Example:
  - File with 2 realizations
    - 0, 50%, 100%
  - File with 5 realizations
    - 0, 20%, 40%, 60%, 80%, 100%
  - File with ii^{th} realizations
    - 100*(1/ii), 100*(2/ii), … 100*(ii-1/ii), 100%

Number of Files	Number of Realizations
23 (unused)	> 3
10 (unused)	3
2 (used)	9



Assessing Stationarity: Non Stationary MNFR

- MNFR stationarity properties
  - mean does not change over time
- Binning data reveals trends in MNFR
- Binning Results
  - MNFR generally decreases over time, but not monotonically, see bins 15 and 38
  - Non-stationary MNFR

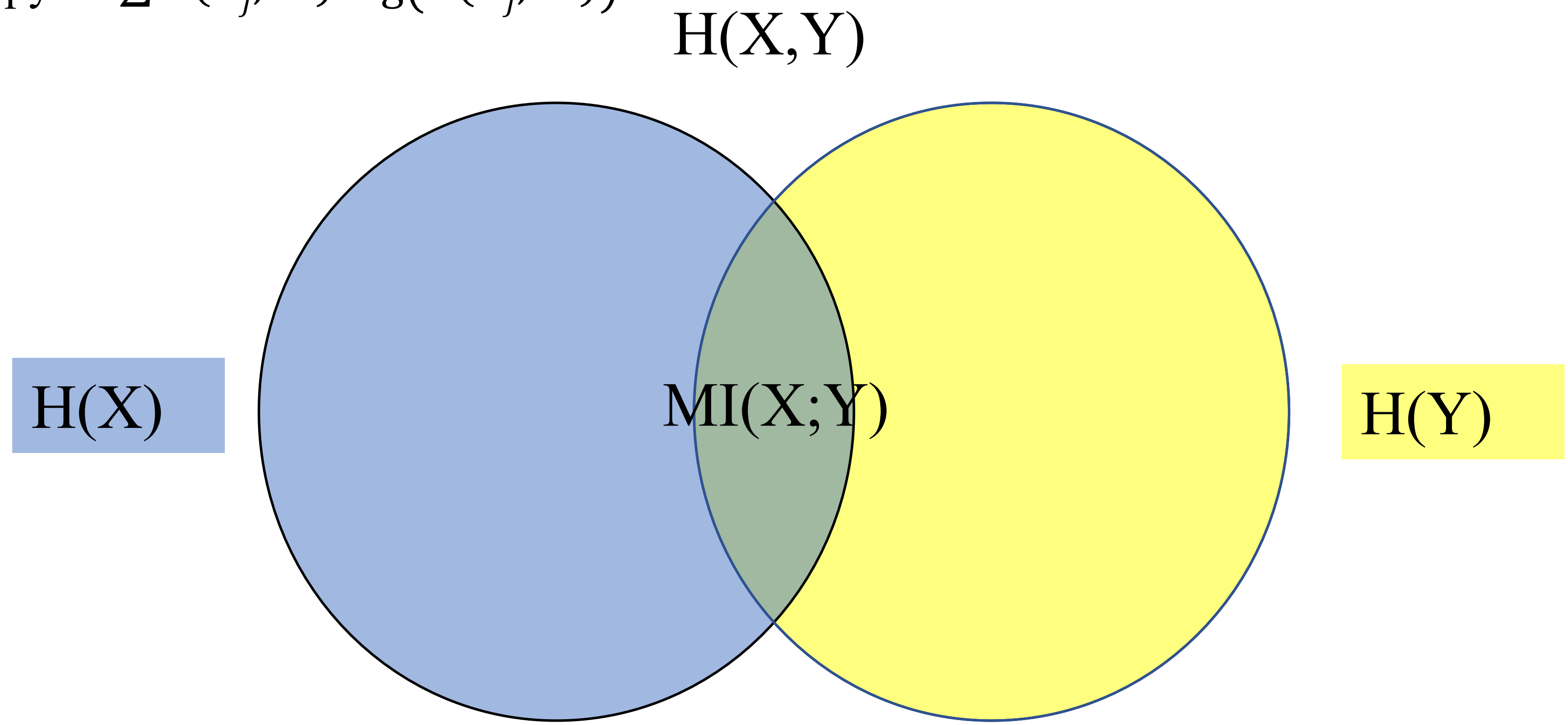


Mutual Information (MI): the Theory

- $MI = H(X) + H(Y) - H(X,Y)$[3]
  - H(X) is the Shannon Entropy: $-\sum P(X_i)\log(P(X_i))$
  - H(Y) is the Shannon Entropy: $-\sum P\,(Y_j)\log(P(Y_j))$
  - H(X,Y) is the Shannon Entropy: $-\sum P(X_j, Yi)\log(P(X_j, Yi))$

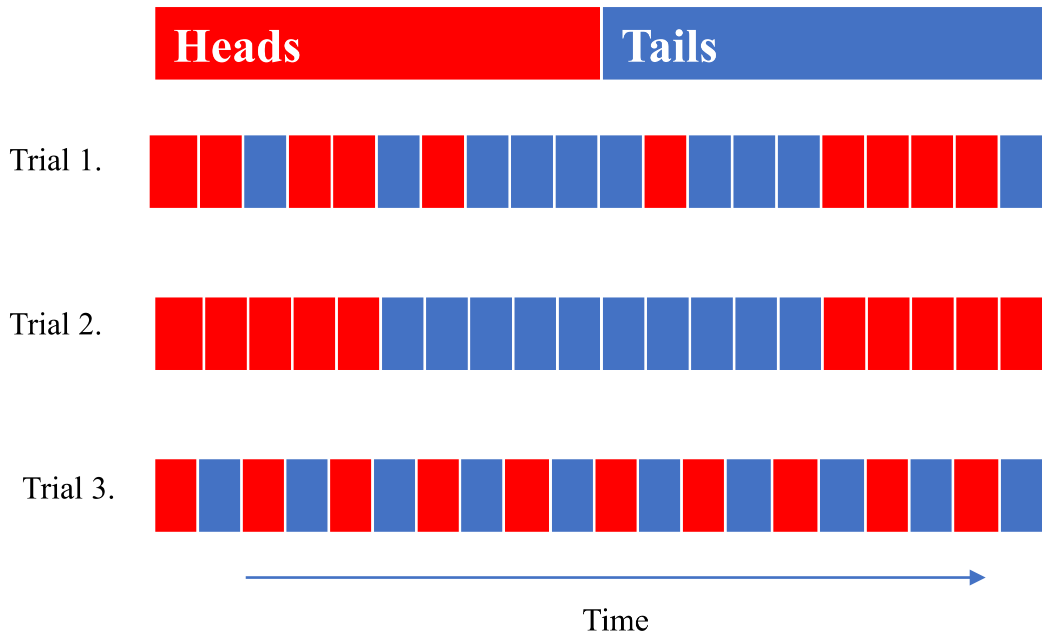
MI qualitatively:

- The amount of common information shared by X and Y
- A measurement of the mutual dependence of two variables



3. Wing, S. Johnson, R. Camporeale, E. Reeves, G. "Information Theoretical Approach to Discovering Solar Wind Drivers of the Outer Radiation Belt." *Journal of Geophysical Research: Space Physics*. 10.1002/2016/JA022711. P. 4.

Mutual Information: An Example



- Flipping a coin experiment
  - Calculating MI for Trial 3. (right)

X

Y

Probabilities: Y / X	Tails	Heads
Heads	1/2	0
Tails	0	1/2

- H(X) = -(1/2)log2(1/2) + -(1/2)log2(1/2) = 1
- H(Y) = -(1/2)log2(1/2) + -(1/2)log2(1/2) = 1
- H(X,Y) = -(1/2)log2(1/2) + -(1/2)log2(1/2) = 1
- MI = 1 + 1 – 1 = 1

	P_{heads}	P_{tails}	$P_{heads, tails}$	$P_{tails, heads}$	$P_{tails, tails}$	$P_{heads, heads}$	MI
Trial 1.	1/2	1/2	5/20	4/20	5/20	5/20	0.008
Trial 2.	1/2	1/2	1/20	1/20	8/20	8/20	0.81
Trial 3.	1/2	1/2	1/2	1/2	0	0	1

Applying Mutual Information

- ISG time series and MI
  - List of order pairs $\{(ISG_1, ISG_{1+\boldsymbol{\tau}}), (ISG_2, ISG_{2+\boldsymbol{\tau}}), \ldots\}$, $\boldsymbol{\tau}$ (tau) = look ahead

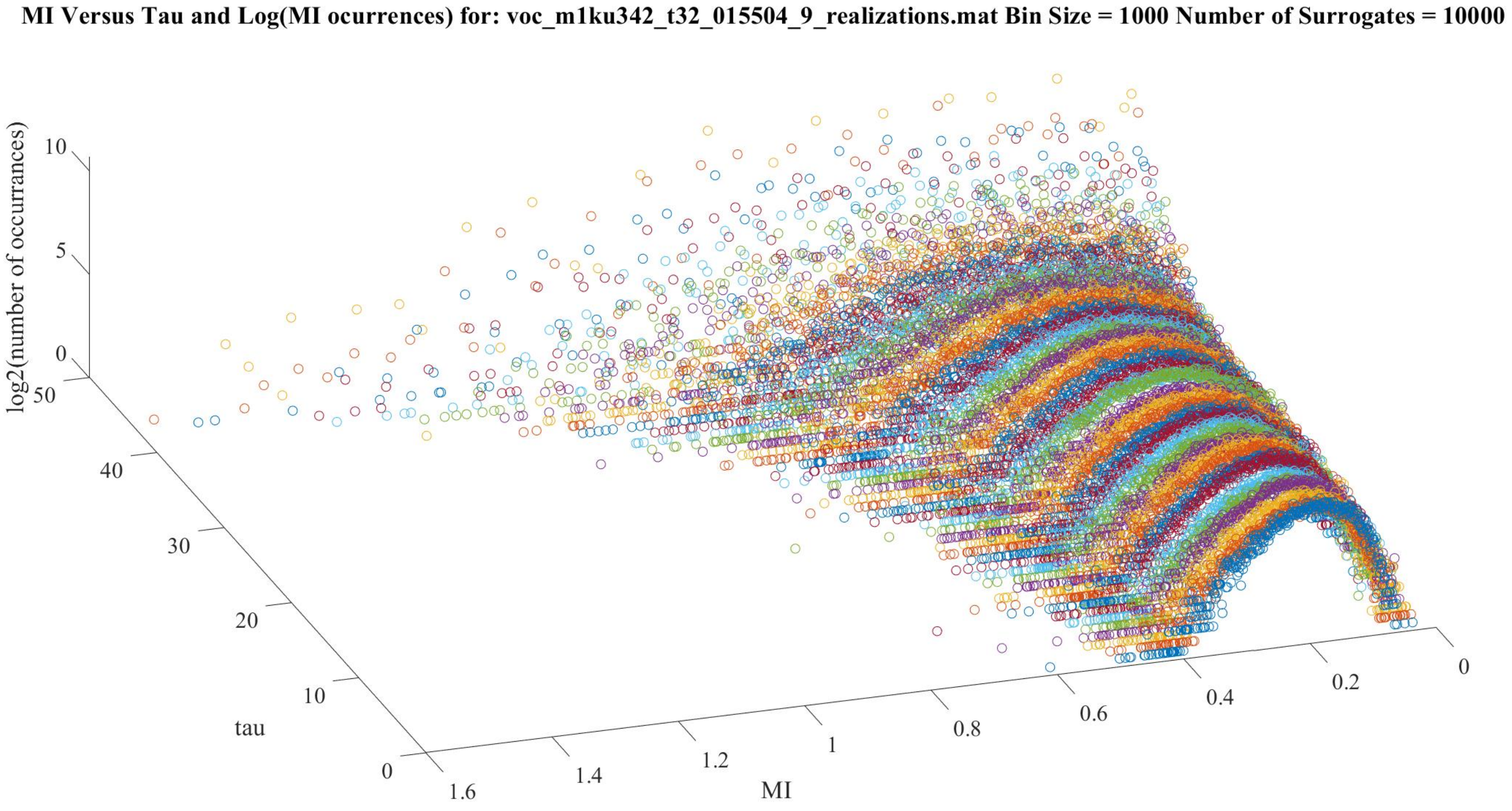
- MI calculation specifics
  - 2D histogram binning generated probability distribution functions (PDF)s / bin
  - Varying $\boldsymbol{\tau}$ showed the distance MI extended into future time

Surrogate Data generation Procedure

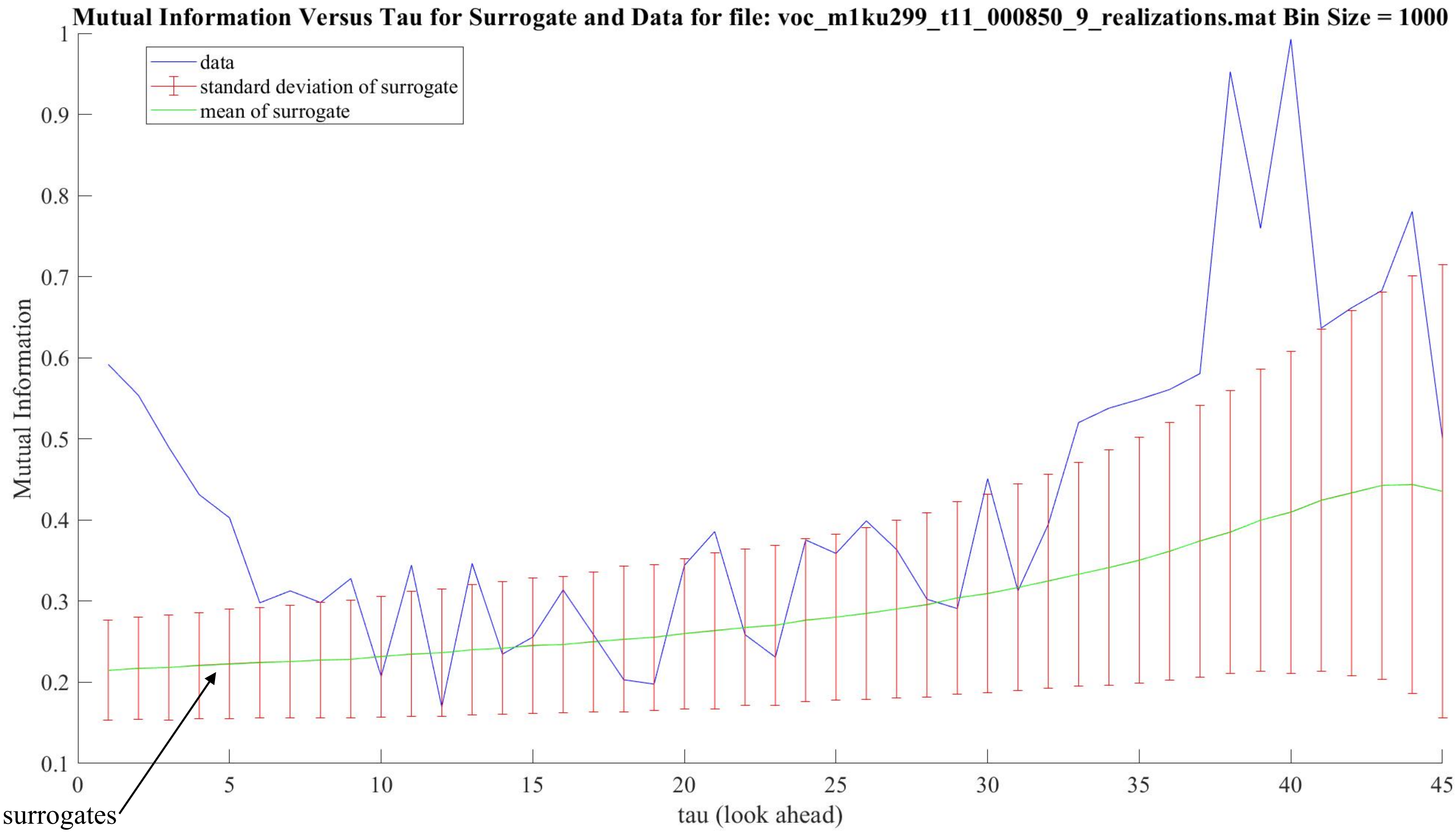
- What is surrogate data?
  - Generated data based off of the MNFR hypothesis that allows for testing MNFR hypothesis with results of actual data
- How do we ensure it is representative of the actual data set?
  - Use MNFR of the data
  - Evenly distributed random values
  - Same MI calculation process as that for actual data
- How many surrogates do we need to generate?
  - 1,000 to 10,000 ISG time series of events
  - When the mean of the MI of the surrogates becomes smooth

Surrogate Data MI results

- 10,000 surrogates
- Normally distributed MI of surrogates
- For comparison with actual data MI, mean of surrogate MI used



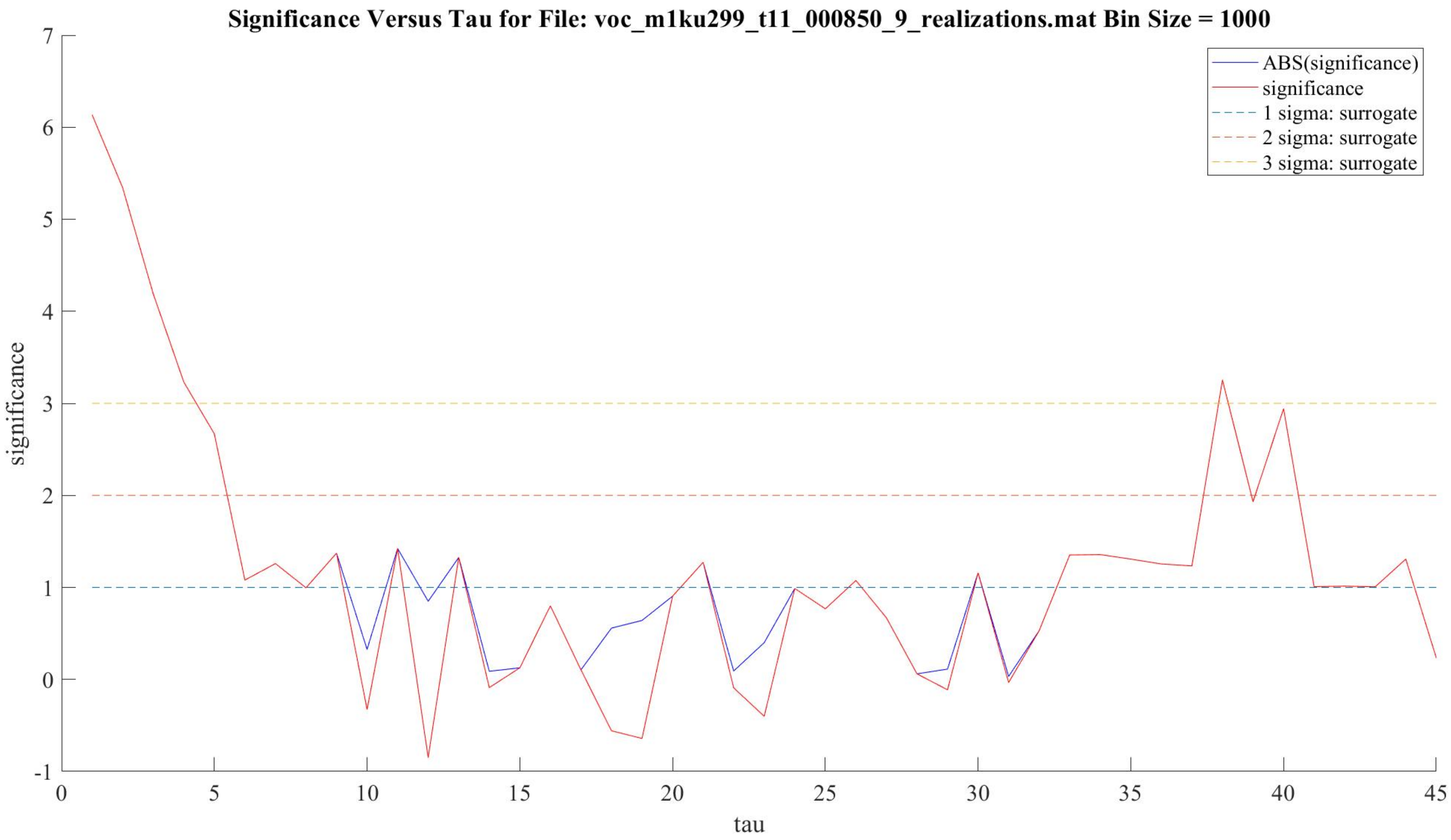
Results: MI versus Look Ahead



Mean of 10,000 surrogates

Results: Significance versus Look Ahead

Significance calculated via $\frac{MI(data)-MI(surr)}{SD(surr)}$, where SD denotes standard deviation and surr denotes surrogate.



Conclusion

- Spikes had significant impact on predicting subsequent spikes at timescales:
  - (0.6 to 0.8 seconds)
  - 0.02 to 0.01) seconds)
- MNFR does not account for all information in considered data set; disproval of null hypothesis
- MATLAB analysis scripts have resulted in development of scalable tool others may deploy in neuron signal analysis

Future Work

- Expanding data set
- Varying realization length and assessing MI at farther out future time
- Increasing number of realizations per file
- Explore relationship of MI and optimal bin size

Acknowledgements and Special Thanks

- **Mentors and Project Contributors:**
  - Dr. Jay R. Johnson, Primary Advisor[1,*]
  - Dr. Ross K. Snider, Data resource[2]

Indirect Funding:

- NASA Grant #: NNX 16AQ87G[*]
- Office of Research and Creative Scholarship, Andrews University[*]

- **Special Thanks to:**
  - Dr. Shanelle M. Henson[1], (surrogate data generation advise)
  - Dr. Giuseppe Passucci[3], (MATLAB instruction)

Seek Knowledge. Affirm Faith. Change the World.

1. Engineering Department, Andrews University ……………………………………………………………......
2. Electrical and Computer Engineering Department, Montana State University……………………………………......
3. SRC Inc. ………………………………………………………………………………………………………

Selected References

[1] Borst, A. Juergen, H. “Effects of Mean Firing on Neural Information Rate.” *Journal of Computational Neuroscience*. 10, 213-221, 2001.

[2] Stein, R. Gossen, R. Jones, K. “Neuronal Variability Noise or Part of the Signal?” *Nature Reviews: Neuroscience*. Vol 6. May 2005. P. 390-7.

[3] Rieke, F. Warland, D. Steeninck, R. Bialek, W. *Spikes Exploring the Neural Code*. MIT Press 1999.

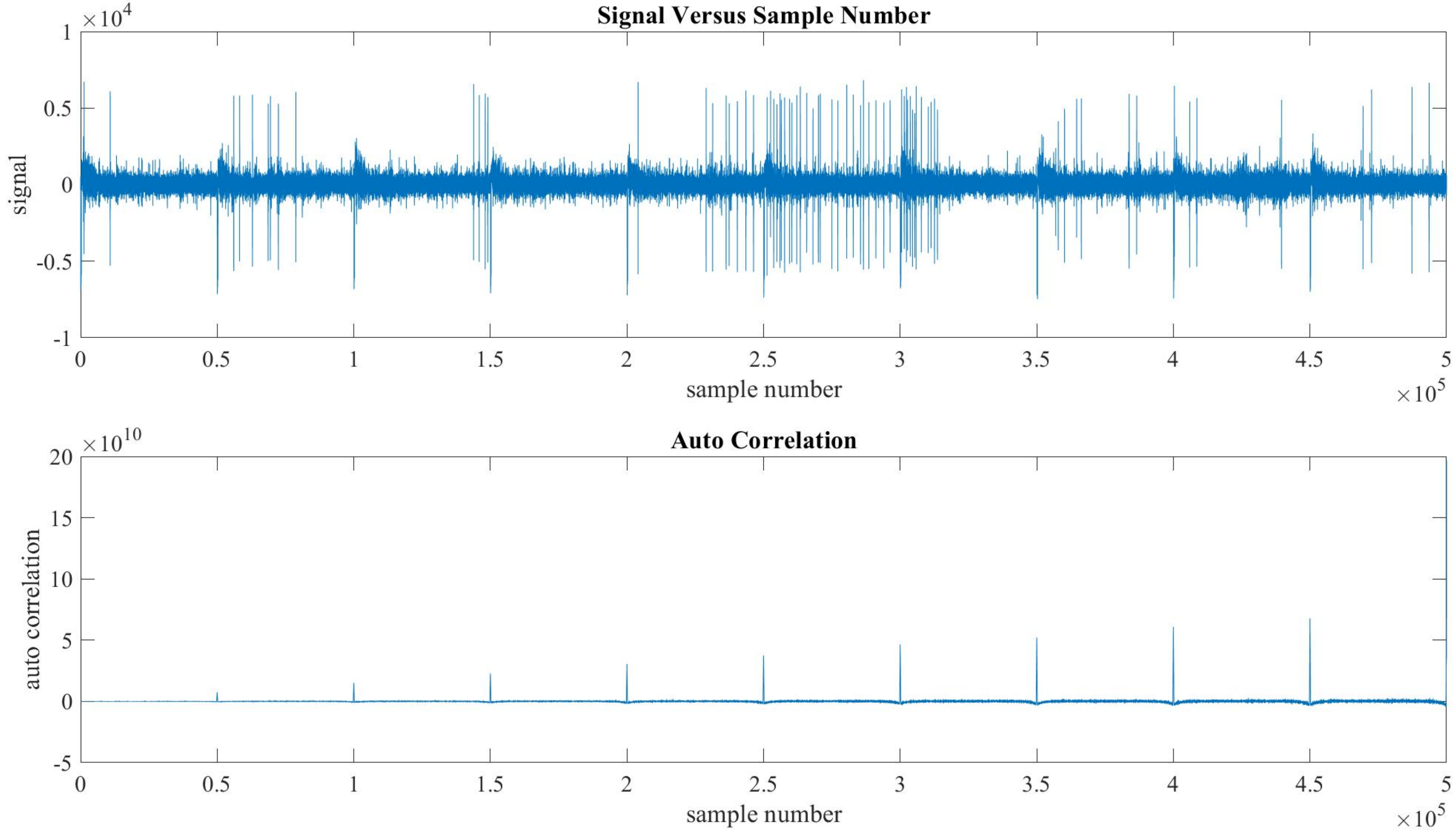
[4] Wing, S. Johnson, R. Camporeale, E. Reeves, G. “Information Theoretical Approach to Discovering Solar Wind Drivers of the Outer Radiation Belt.” *Journal of Geophysical Research: Space Physics*. 10.1002/2016/JA022711. P. 4.

Questions Comments?

Starting backup Slides

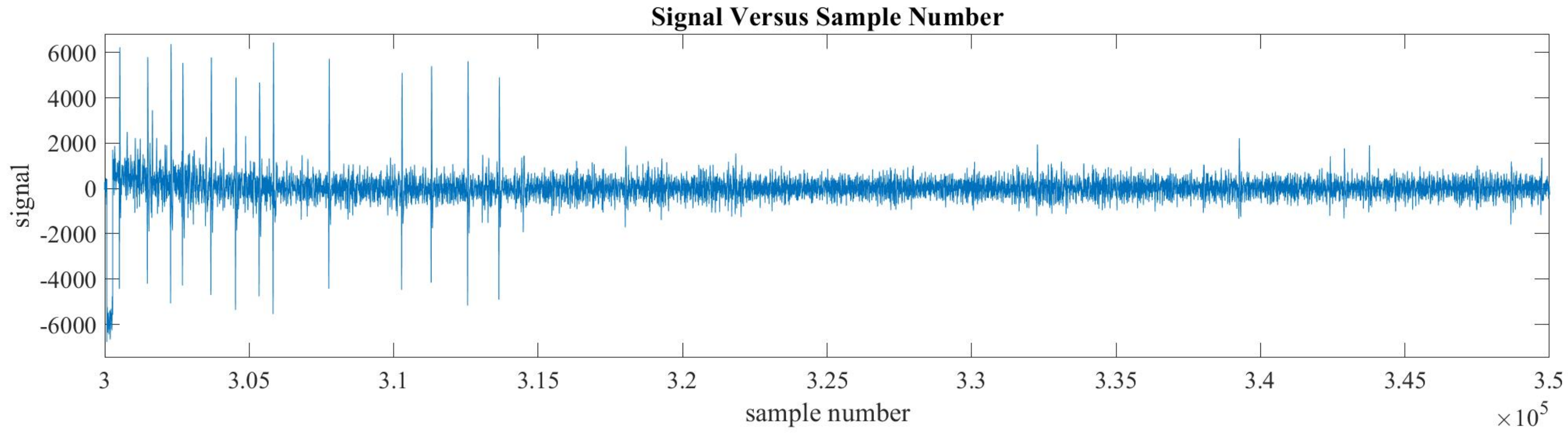
- Which bin size to use?
  - Range of bin sizes
    - Upper bound = MNFR (1/1670 [spikes / sample])
    - Lower bound = smallest ISG (1/300 [spikes / sample])
      - Determined by brain processing speed-note maximum processing frequency 10-12 Hz, car wheels spin backwards above this frequency
  - Bin size used = mean of upper and lower bounds = 1000 samples
  - Bin size chosen to optimize PDF of spikes / bin distribution

Auto Correlation Comparison



Auto Correlation Comparison

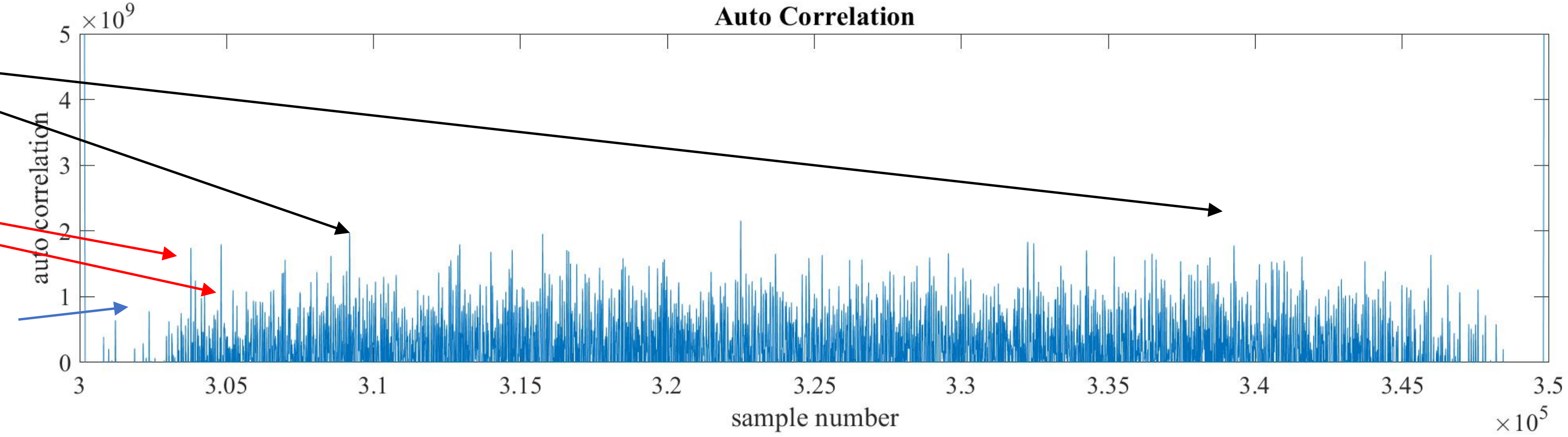
- One realization auto correlation



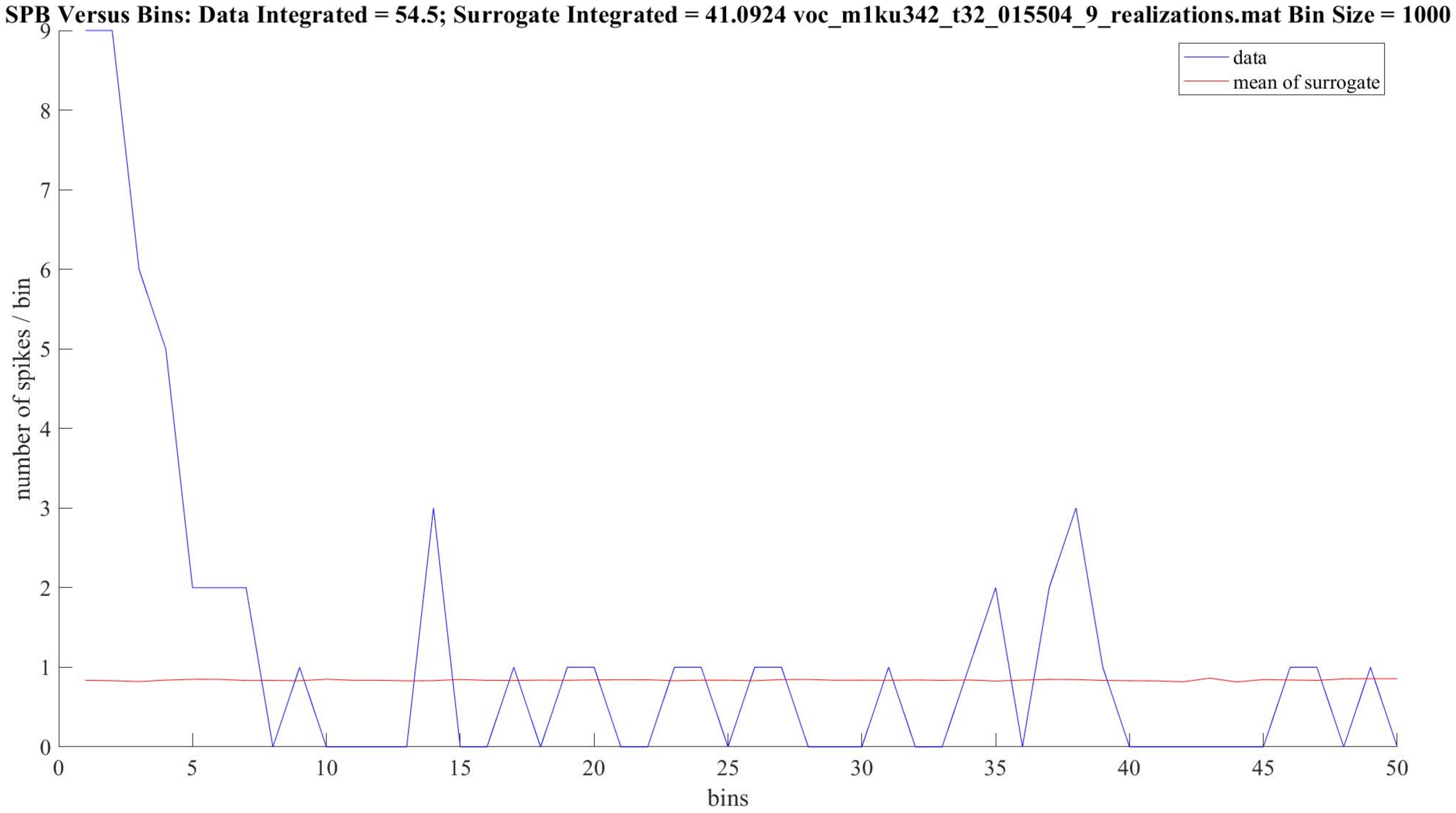
~ 30,000 samples gap

~ 500 samples gap

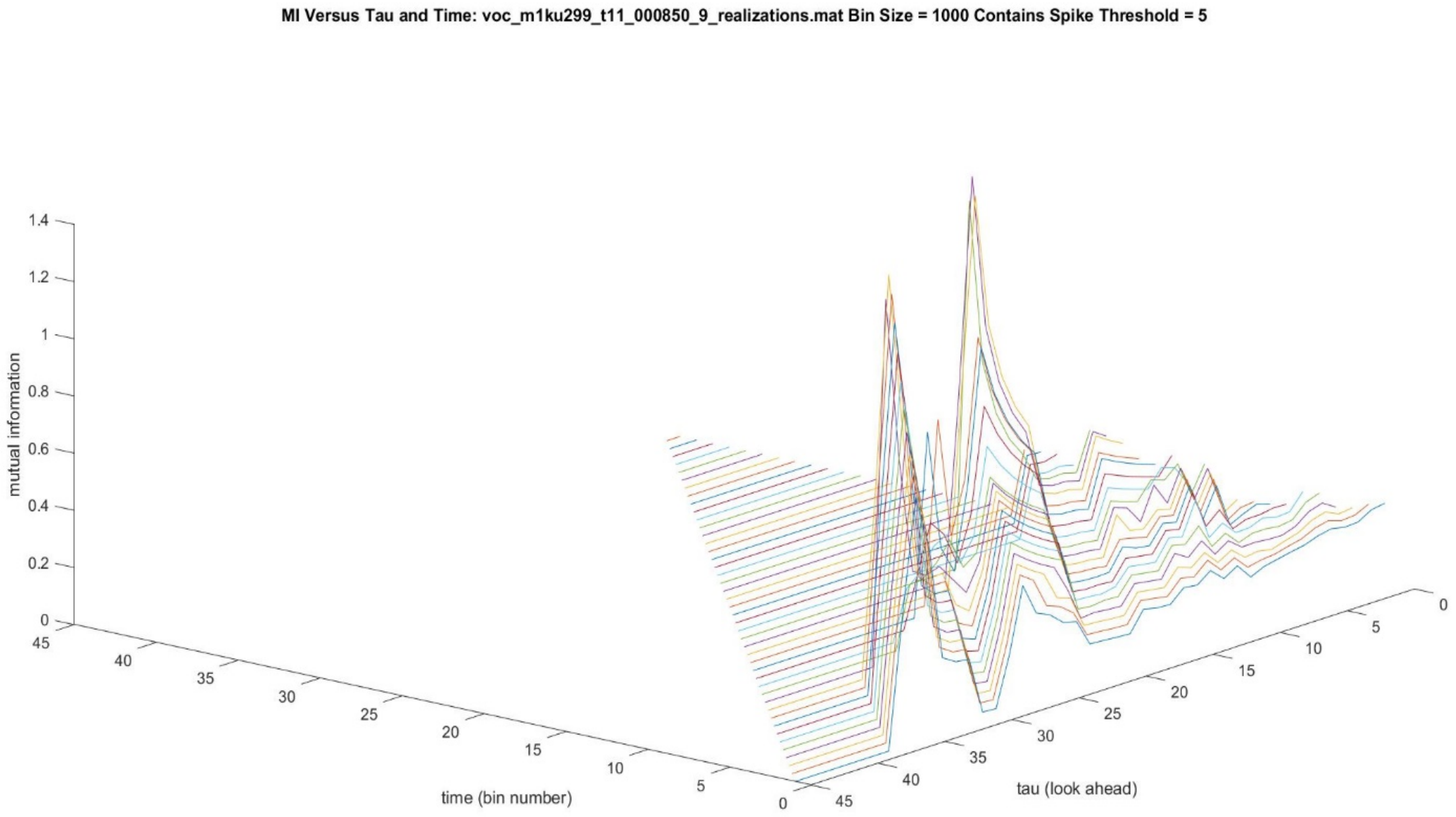
~ 1670 samples / spike MNFR



Comparison of Spikes Per Bin (SPB) of Surrogate and Data

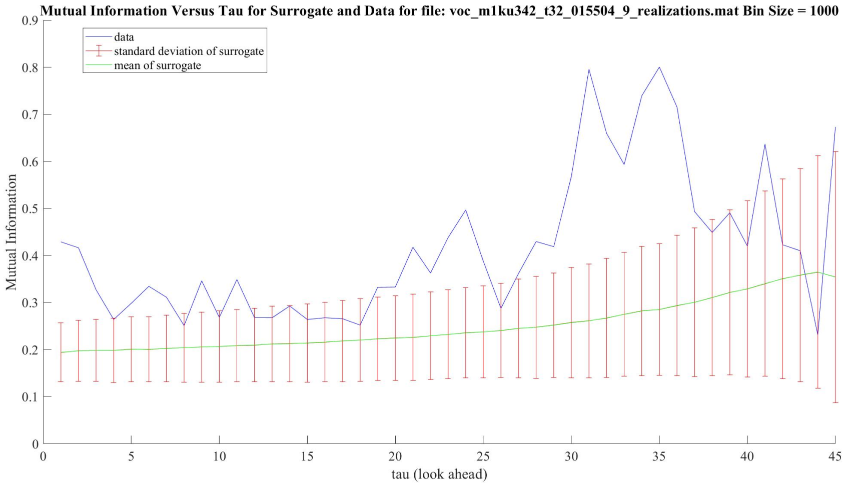


MI versus Time versus Tau

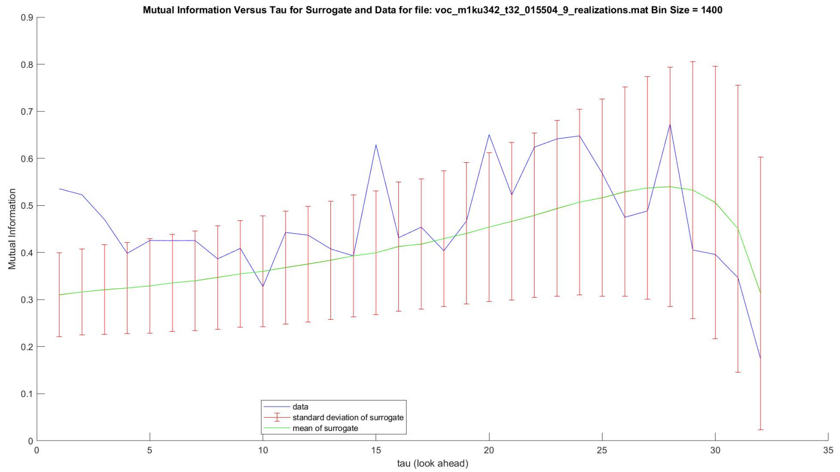


MI versus Tau for Various Bin Sizes

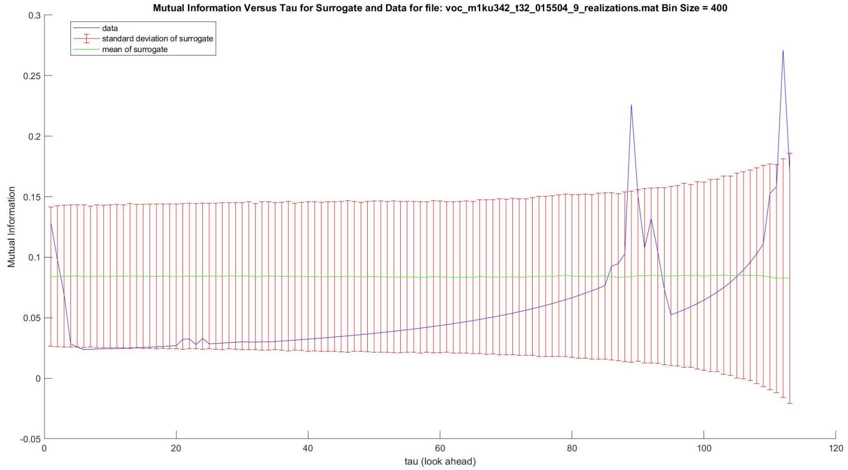
Bin size = 1000 samples



Bin size = 1400 samples



Bin size = 400 samples



Bin size = 2400 samples